SEC 1 MATH CHEAT SHEET

Content:

REAL NUMBERS

Section A: Classification of Real Numbers

Real Numbers

Any number found in the real world

- **Rational Numbers**

  Numbers that can be expressed as $\frac{a}{b}$ where a and b are integers and $b \neq 0$ e.g. 0.5, -0.45, 7.232323

  - Integers
    - **Whole Numbers**

      E.g. 0,1,2,3

      - 0
      - **Positive Integers / Natural Numbers**

        E.g. 1,2,3,4

        - 1
        - **Composite Numbers**

          Numbers that have more than 2 factors (Numbers that can be divided by other numbers apart from 1 and itself) e.g. 4,6,8,9,10,12,14,16
        - **Prime Numbers**

          Numbers that have only 2 factors which are 1 and itself (Numbers that can be divided by 1 and itself only) e.g. 2,3,5,7,11,13,17
    - **Negative Integers**

      E.g. -1,-2,-3
  - Fractions
    - **Recurring Decimal**

      A decimal that has a digit/group of digits that repeats e.g. 0.3, $9.\overline{25}$
    - **Terminating Decimal**

      E.g. 0.125, 0.5, 0.25

- **Irrational Numbers**

  Decimals that are non-terminating & non-recurring e.g. 3.141592, 3.9600559..

Arrange the following in descending order.

$-\frac{1}{3}, -\pi, -\frac{1}{\sqrt{11}}, \sqrt[3]{0.05}, -0.3, -\frac{7}{20}$

Step 1: Separate the positive and negative numbers. List the numbers in decimals.

$\sqrt[3]{0.05}$ = 0.36840 ...

$-\frac{1}{3}$ = 0.33333 ...

$-\pi$ = - 3.1415 ...

$-\frac{1}{\sqrt{11}}$ = - 0.30151 ...

- 0.3 = - 0.3

$-\frac{7}{20}$ = - 0.35

Listing the numbers in decimals makes comparison easier.

Step 2: Sort the positive and negative numbers separately and align the place values. Compare the digits of the numbers from the left most place value. In this case, we start comparing from the ones place, followed by the tenths, then hundredths, etc.

$\sqrt[3]{0.05}$ = 0.36840 ... (4)

$-\frac{1}{3}$ = 0.33333 ... (1)

$-\pi$ = - 3.1415 ... (6)

$-\frac{1}{\sqrt{11}}$ = - 0.30151 ... (3)

- 0.3 = - 0.3 (2)

$-\frac{7}{20}$ = - 0.35 (5)

For negative numbers, the bigger the number (without the negative sign), the smaller it is (with the negative sign). E.g. comparing -0.33333 ... and -0.30151 ..., if we look at the numbers without their negative sign, 0.33333 ... is larger than 0.30151 ... . For negative numbers, the opposite is true, -0.30151 ... is larger than -0.33333 ...

Step 3: List the numbers from the question in descending order.

$\sqrt[3]{0.05}, -0.3, -\frac{1}{\sqrt{11}}, -\frac{1}{3}, -\frac{7}{20}, -\pi$

Section B: Order of Operation

B	**Brackets**	From innermost bracket to outermost bracket, () [] {}
O	**Order**	Indices, powers, and roots, $x^2 \sqrt{x}$
D	**Division**	Performed concurrently, from left to right
M	**Multiplication**	
A	**Addition**	Performed concurrently, from left to right
S	**Subtraction**	

Section C: Application

The table shows the temperature of four cities at noon on a particular day in December.

Singapore	**27°C**
Shanghai	**4.6°C**
Hong Kong	**16.3°C**
Mongolia	**-21.6°C**

a) Find the difference in temperature between Singapore and Mongolia.

27℃ - (-21.6℃) = 27℃ + 21.6℃

= 48.6℃

Difference means "—"

b) Find the average temperature of the 4 cities.

$$\text{Average} = \frac{27℃ + 4.6℃ + 16.3℃ + (-21.6℃)}{4}$$

= 6.575℃

$$\text{Average} = \frac{\text{sum of all values}}{\text{total number of values}}$$

c) The temperature in Saudi Arabia is midway between the temperature in Shanghai and Hong Kong. What is the temperature in Saudi Arabia?

$$\text{Temperature in Saudi Arabia} = \frac{4.6℃ + 16.3℃}{2}$$

= 10.45℃

$$\text{Midpoint} = \frac{\text{sum of all values}}{2}$$

d) If the temperature in Shanghai drops by 5.5℃ one month later. Find Shanghai's temperature in one month's time.

Temperature in one month's time = 4.6℃ - 5.5℃

= -0.9℃

PRIME NUMBER, HCF, LCM

Section A: Prime Factorisation

Positive Integers / Natural Numbers

E.g. 1,2,3,4

1

Composite Numbers

Numbers that have more than 2 factors (Numbers that can be divided by other numbers apart from 1 and itself) e.g. 4,6,8,9,10,12,14,15

Prime Numbers

Numbers that have only 2 factors which are 1 and itself (Numbers that can be divided by 1 and itself only) e.g. 2,3,5,7,11,13,17

Smallest prime number is 2!

List of prime numbers between 1 to 30:

1	2	3	4	5
6	7	8	9	10
11	12	13	14	15
16	17	18	19	20
21	22	23	24	25
26	27	28	29	30

Express 4840 in index notation.

2	4840
2	2420
2	1210
5	605
11	121
11	11
	1

Only use prime numbers to prime factorise 4840

$4840 = 2^3 \times 5 \times 11^2$

Check your answer.
Using calculator,
$2^3 \times 5 \times 11^2 = 4840$

Section B: Highest Common Factor (HCF)

Find the highest common factor of the following numbers in index notation.

$924 = 2^2 \times 3 \times 7 \times 11$
$2520 = 2^3 \times 3^2 \times 5 \times 7$
$2548 = 2^2 \times 7^2 \times 13$

Step 1: Align the bases for easy comparison.

$924 \ \ = 2^2 \times 3 \qquad \times 7 \times 11$
$2520 = 2^3 \times 3^2 \times 5 \times 7$
$2548 = 2^2 \qquad\qquad \times 7^2 \qquad \times 13$

Step 2: To find HCF, only take **common** bases of ALL the numbers, 924, 2520, and 2548.

$924 \ \ = 2^2 \times 3 \qquad \times 7 \times 11$
$2520 = 2^3 \times 3^2 \times 5 \times 7$
$2548 = 2^2 \qquad\qquad \times 7^2 \qquad \times 13$

HCF $= 2 \qquad\qquad \times 7$

HCF is **STINGY**!

1) Bases: take **common/repeated** bases of all numbers.
2) Powers: take **lowest** power

Step 3: Take the **lowest** power of the prime numbers listed in Step 2.

$924 \ \ = 2^2 \times 3 \qquad \times 7^1 \times 11$
$2520 = 2^3 \times 3^2 \times 5 \times 7^1$
$2548 = 2^2 \qquad\qquad \times 7^2 \qquad \times 13$

2 1

HCF $= 2^2 \qquad\qquad \times 7^1$

Do not calculate out the index notation if question is asking for "index notation"

Check your answer.
2 x 7 = 28
Since 28 is the highest factor of 924, 2520 and 2548, all three numbers should be divisible by 28.

$\frac{924}{28} = 33$

$\frac{2520}{28} = 90$

$\frac{2548}{28} = 91$

Section C: Lowest Common Multiple (LCM)

Find the lowest common multiple of the following numbers.

$924 = 2^2 \times 3 \times 7 \times 11$
$2520 = 2^3 \times 3^2 \times 5 \times 7$
$2548 = 2^2 \times 7^2 \times 13$

Step 1: Align the bases for easy comparison.

$$\begin{aligned} 924 &= 2^2 \times 3 \quad\quad \times 7 \times 11 \\ 2520 &= 2^3 \times 3^2 \times 5 \times 7 \\ 2548 &= 2^2 \quad\quad\quad\quad \times 7^2 \quad \times 13 \end{aligned}$$

Step 2: To find LCM, only take **all** bases of ALL the numbers, 924, 2520, and 2548.

$$\begin{aligned} 924 &= 2^2 \times 3 \quad\quad \times 7 \times 11 \\ 2520 &= 2^3 \times 3^2 \times 5 \times 7 \\ 2548 &= 2^2 \quad\quad\quad\quad \times 7^2 \quad \times 13 \\ \hline \text{LCM} &= 2 \times 3 \times 5 \times 7 \times 11 \times 13 \end{aligned}$$

LCM is **GREEDY**!

1) Bases: take **all** bases of all numbers.
2) Powers: take **highest** power

Step 3: Take the **highest** power of the prime numbers listed in Step 2.

$$\begin{aligned} 924 &= 2^2 \times 3 \quad\quad \times 7 \times 11 \\ 2520 &= 2^3 \times 3^2 \times 5 \times 7 \\ 2548 &= 2^2 \quad\quad\quad\quad \times 7^2 \quad \times 13 \\ \hline \text{LCM} &= 2^3 \times 3^2 \times 5 \times 7^2 \times 11 \times 13 \\ &= 2522520 \end{aligned}$$

Do not calculate out the index notation if question is asking for "index notation"

Check your answer.
Since 2522520 is the lowest multiple of 924, 2520 and 2548, 2522520 should be divisible by all three numbers.

$\frac{2522520}{924}$ = 2730 (2522520 is the 2730th multiple of 924)

$\frac{2522520}{2520}$ = 1001 (2522520 is the 1001st multiple of 2520)

$\frac{2522520}{2548}$ = 990 (2522520 is the 990th multiple of 2548)

Section D: Perfect Squares

Find the smallest value of *k* such that 2520*k* is a perfect square. OR
Find the smallest value of *k* such that $\sqrt{2520k}$ is a whole number.

$2520 = 2^3 \times 3^2 \times 5 \times 7$

$\sqrt{2520} = \sqrt{2^3 \times 3^2 \times 5^1 \times 7^1 \times k}$ ⤸ $\times (2 \times 5 \times 7)$

$= \sqrt{2^4 \times 3^2 \times 5^5 \times 7^2}$

$\therefore k = 2 \times 5 \times 7$
$= 70$

For perfect square, powers must be a multiple of 2 / divisible by 2.

If the number, 2520, is <u>multiplied</u> by *k*, you can only <u>increase</u> the power of the prime factors to the next smallest multiple of 2.

Check your answer.

$\sqrt{2520 \times 70} = 420$

Since 420 is a whole number, $\sqrt{2520k}$ is a perfect square.

Find the smallest value of *k* such that $\frac{2520}{k}$ is a perfect square. OR

Find the smallest value of *k* such that $\sqrt{\frac{2520}{k}}$ is a whole number.

$2520 = 2^3 \times 3^2 \times 5 \times 7$

$\sqrt{\frac{2520}{k}} = \sqrt{\frac{2^3 \times 3^2 \times 5^1 \times 7^1}{k}}$ ⤸ $\div (2 \times 5 \times 7)$

$= \sqrt{\frac{2^3 \times 3^2 \times 5^1 \times 7^1}{2 \times 5 \times 7}}$

$= \sqrt{2^2 \times 3^2}$

$\therefore k = 2 \times 5 \times 7$
$= 70$

If the number, 2520, is divided by *k*, you can only decrease the power of the prime factors to the next biggest multiple of 2, or to 0 to completely remove the base.

Check your answer.

$\sqrt{2520 \times 70} = 420$

Since 420 is a whole number, $\sqrt{2520k}$ is a perfect square.

Section E: Perfect Cubes

Find the smallest value of *k* such that 2520*k* is a perfect cube. OR
Find the smallest value of *k* such that $\sqrt[3]{2520k}$ is a whole number.

$2520 = 2^3 \times 3^2 \times 5 \times 7$

$\sqrt[3]{2520} = \sqrt[3]{2^3 \times 3^2 \times 5^1 \times 7^1 \times k}$ ⟶ $\times (3 \times 5^2 \times 7^2)$

$= \sqrt{2^3 \times 3^3 \times 5^3 \times 7^3}$

∴ $k = 3 \times 5^2 \times 7^2$
$= 3675$

Check your answer.

$\sqrt[3]{2520 \times 3675} = 210$

Since 210 is a whole number, $\sqrt[3]{2520k}$ is a perfect cube.

For perfect cubes, powers must be a multiple of 3 or divisible by 3.

If the number, 2520, is <u>divided</u> by *k*, you can only <u>increase</u> the power of the prime factors to the next smallest multiple of 3.

Find the smallest value of *k* such that $\frac{2520}{k}$ is a perfect cube. OR

Find the smallest value of *k* such that $\sqrt[3]{\frac{2520}{k}}$ is a whole number.

$2520 = 2^3 \times 3^2 \times 5 \times 7$

$\sqrt[3]{\frac{2520}{k}} = \sqrt[3]{\frac{2^3 \times 3^2 \times 5^1 \times 7^1}{k}}$ ⟶ $\div (3^2 \times 5 \times 7)$

$= \sqrt{\frac{2^3 \times 3^2 \times 5^1 \times 7^1}{3^2 \times 5 \times 7}}$

$= \sqrt{2^3}$

∴ $k = 3^2 \times 5 \times 7$
$= 315$

If the number, 2520, is <u>divided</u> by *k*, you can only <u>decrease</u> the power of the prime factors to the next biggest multiple of 3, or to 0 to completely remove the base.

$\sqrt{\frac{2^3 \times \cancel{3^2}^{1} \times \cancel{5^1}^{1} \times \cancel{7^1}^{1}}{{}_1\cancel{3^2} \times {}_1\cancel{5} \times {}_1\cancel{7}}} = \sqrt{\frac{2^3 \times 1 \times 1 \times 1}{1 \times 1 \times 1}}$

Check your answer.

$\sqrt[3]{\frac{2520}{315}} = 2$

Since 2 is a whole number, $\sqrt[3]{\frac{2520}{k}}$ is a perfect cube.

Section F: Application

Find the smallest positive integer *k* such that 392*k* is a multiple of 396.

$392 = 2^3 \times 7^2$

$396 = 2^2 \times 3^2 \times 11$

$$\frac{392k}{396} = \frac{2^3 \times 7^2 \times k}{2^2 \times 3^2 \times 11}$$

$$= \frac{2 \times 7^2 \times k}{3^2 \times 11}$$

$\times (3^2 \times 11)$

$$= \frac{2 \times 7^2 \times 3^2 \times 11}{3^2 \times 11}$$

$$= \frac{2 \times 7^2}{1}$$

$\therefore k = 3^2 \times 11$

$= 99$

Simplify the fraction

$$\frac{2^{3\,1} \times 7^1 \times k}{{}_1 2^2 \times 3^2 \times 11} = \frac{2 \times 7^2 \times k}{3^2 \times 11}$$

k = denominator

For 392*k* to be a multiple of 396, $\frac{392k}{396}$ must leave no remainder or denominator must be reduced to 1.

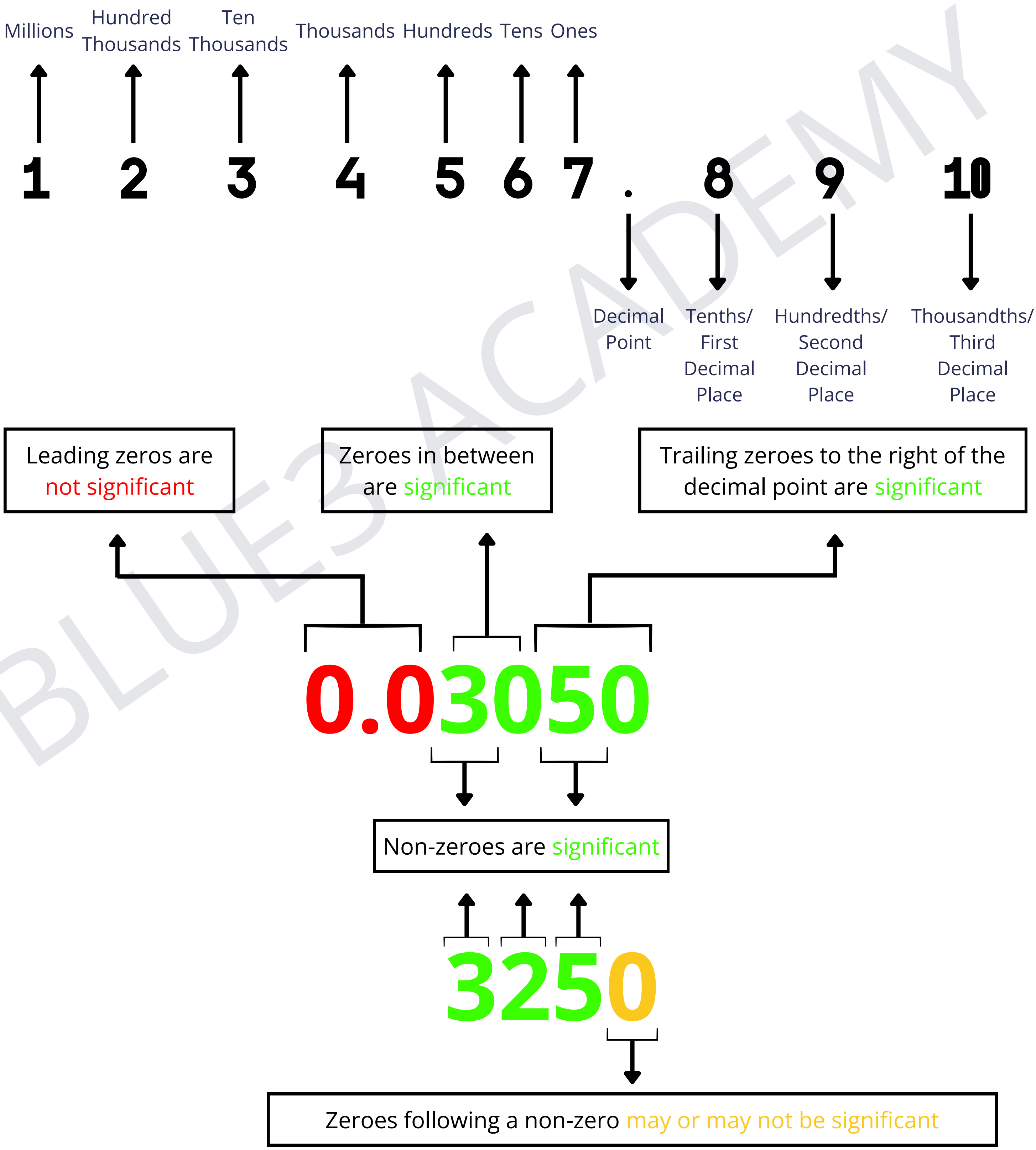
Check your answer.
Using calculator,

$$\frac{392 \times 99}{396} = 98$$

Since 98 is a whole number, $\frac{392k}{396}$ is the 98th multiple of 396.

ESTIMATION & APPROXIMATION

Section A: Rounding Off



Section B: Estimation

By rounding each number to 1 significant figure, estimate $\frac{460 \times 12.95}{12.3 \times 4.6}$

$$\frac{460 \times 12.95}{12.3 \times 4.6} \approx \frac{460 \times 12.95}{12.3 \times 4.6}$$

$$= \frac{500 \times 10}{10 \times 5}$$

$$= 100$$

Use ≈ when estimating numbers

Check your answer.

$$\frac{460 \times 12.95}{12.3 \times 4.6} = 105\frac{35}{123}$$

$$= 100 \ (1sf)$$

Estimate the value of $34.5 \times \frac{6.03}{\sqrt[3]{28.1}}$, giving your answer correct to 1 significant figure.

$$34.5 \times \frac{6.03}{\sqrt[3]{28.1}} \approx 35 \times \frac{6.0}{\sqrt[3]{27}}$$

$$= \frac{210}{3.0}$$

$$= 70 \ (1sf)$$

Remember to correct your answer to 1sf

In an estimation question, when the question states should be corrected to a certain degree of accuracy, working should be presented in 1 degree higher.

E.g. if the question states to "give your answer correct to 1sf", working should be estimated to 2sf.

Check your answer.

$$34.5 \times \frac{6.03}{\sqrt[3]{28.1}} = 68.42807151$$

$$= 70 \ (1sf)$$

Estimate numbers in square roots to the closest perfect square number and numbers in cube roots to the closest perfect cube numbers

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